Image-based Lighting with a Piecewise-Constant Importance Function

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Overview

- HDRI lighting as an integral
- The beauty of importance sampling
- Importance sampling for HDRI lighting
- Pretty Pictures

HDRI Lighting

The reflected radiance calculation for direct illumination

$$L_r(p, e) = \int_{\omega \in \Omega} L(p, \omega) f_r(\omega, n, e) V_p(\omega)(\omega \cdot n) d\omega$$

HDRI Lighting

The reflected radiance calculation for direct illumination

$$L_r(p,e) = \int_{\omega\in\Omega} L(p,\omega) f_r(\omega,n,e) V_p(\omega)(\omega\cdot n)\,d\omega$$

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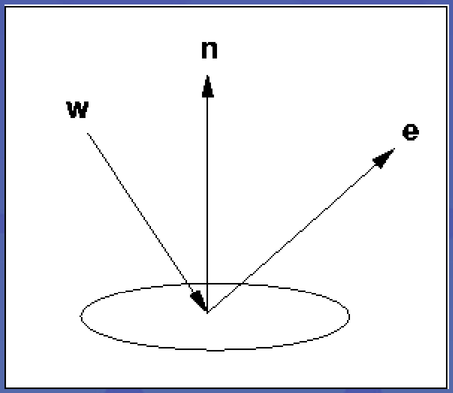
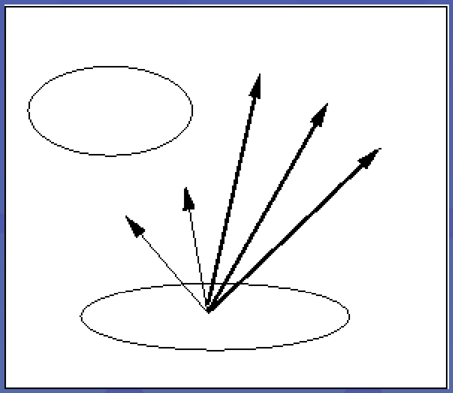


HDRI Lighting

The reflected radiance calculation for direct illumination

$$L_r(p,e) = \int_{\omega \in \Omega} L(p,\omega) f_r(\omega,n,e) V_p(\omega) (\omega \cdot n) d\omega$$



HDRI Lighting

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HDRI Lighting

The reflected radiance calculation for direct illumination

$$L_r(p,e) = \int_{\omega\in\Omega} L(p,\omega) f_r(\omega,n,e) V_p(\omega)(\omega\cdot n)\,d\omega$$

Let’s simplify:

$$L_r(p,e) = \int_{\omega\in\Omega}$$

HDRI Lighting

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Let's simplify:

$$L_r(p,e) = \int_{\omega\in\Omega} L(\omega)$$

HDRI Lighting

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$$L_r(p,e) = \int_{\omega\in\Omega} L(p,\omega) f_r(\omega,n,e) V_p(\omega)(\omega\cdot n)d\omega$$

Let's simplify:

$$L_r(p,e) = \int_{\omega\in\Omega} L(\omega)\cdot 1$$

HDRI Lighting

The reflected radiance calculation for direct illumination

$$L_r(p,e) = \int_{\omega \in \Omega} L(p,\omega) f_r(\omega, n, e) V_p(\omega)(\omega \cdot n) d\omega$$

Let's simplify:

$$L_r(p) = \int_{\omega \in \Omega} L(\omega) \cdot 1$$

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Let's simplify:

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HDRI Lighting

The reflected radiance calculation for direct illumination

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Let's simplify:

$$L_r(p) = \int_{\omega \in \Omega} L(\omega) V_p(\omega)(\omega \cdot n) d\omega$$

HDRI Lighting

Rewrite as an integral of a simple function.

$$H(\omega, p, n) = L(\omega)V_p(\omega)(\omega \cdot n)$$

$$L_r(p) = \int_{\omega \in \Omega} H(\omega, p, n)d\omega$$

HDRI Lighting

HDRI Lighting as an integral

- For each pixel
- Cast a ray to find p and n
- Evaluate $\int_{\omega \in \Omega} H(\omega, p, n) d\omega$

Importance Sampling

Choose an *importance function* $I(\omega)$ with

$$\int_{\omega\in\Omega} I(\omega)d\omega = 1$$

Draw samples $\omega_i \sim I(\omega)$

$$\int_{\omega\in\Omega} H(\omega)d\omega \approx \frac{1}{N}\sum_{i=1}^{N}\frac{H(\omega_i)}{I(\omega_i)}$$

Importance Sampling

Let $I(\omega) \sim H(\omega) \Rightarrow$

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Importance Sampling

$$\text{Let } I(\omega) \sim H(\omega) \Rightarrow$$

$$I = \alpha H \Rightarrow$$

$$\int I d\omega = \alpha \int H d\omega \Rightarrow$$

$$1 = \alpha \int H d\omega \Rightarrow$$

$$\alpha = \left(\int H d\omega \right)^{-1}.$$

Importance Sampling

$$\frac{1}{N}\sum_{i=1}^{N}\frac{H(\omega_i)}{I(\omega_i)} = \frac{1}{N}\sum_{i=1}^{N}\frac{H(\omega_i)}{\alpha H(\omega_i)} =$$

$$\frac{1}{N}\sum_{i=1}^{N}\frac{1}{\alpha} = \frac{1}{\alpha} = \int_{\omega\in\Omega} H(\omega)d\omega.$$

TAKE AWAY: We get exact answer *regardless* of N!

(When I is *almost* proportional to H,
convergence rate improves).

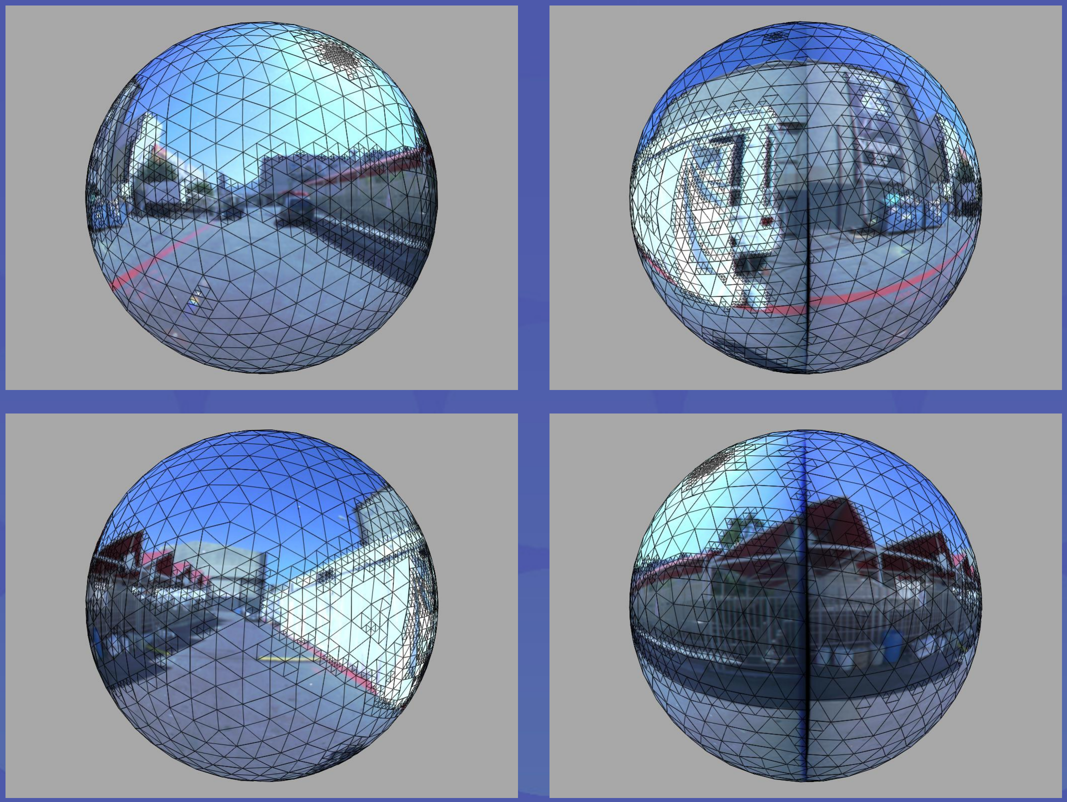
Importance Sampling for HDRI Lighting

- Find an approximation to H, $\hat{H}$
- Use $\hat{H}$ as importance function
- Better approximation $\Rightarrow$ sampling will converge faster
- $\hat{H}$ is a family of functions (one per surface normal n)
- $\hat{H}$ must be easy to evalulate and sample from

Piecewise Constant Importance Sampling

Idea: Use piecewise constant function

Subdivide Env Map into triangles

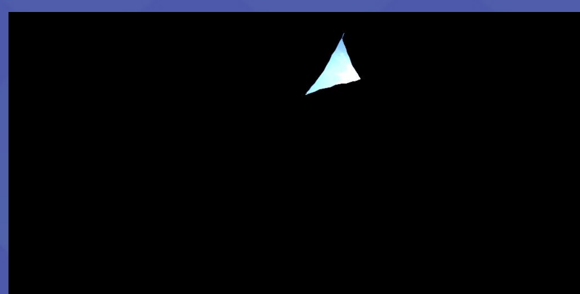


Preconvolution

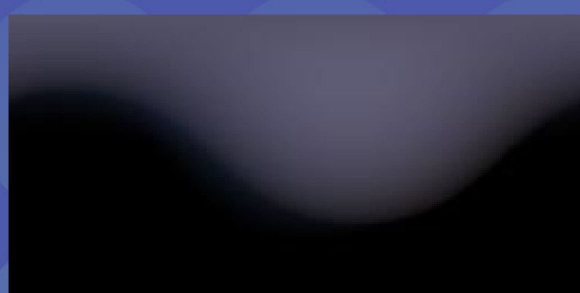
For each triangle:

- Store total irradiance as a function of surface normal.

Original Image

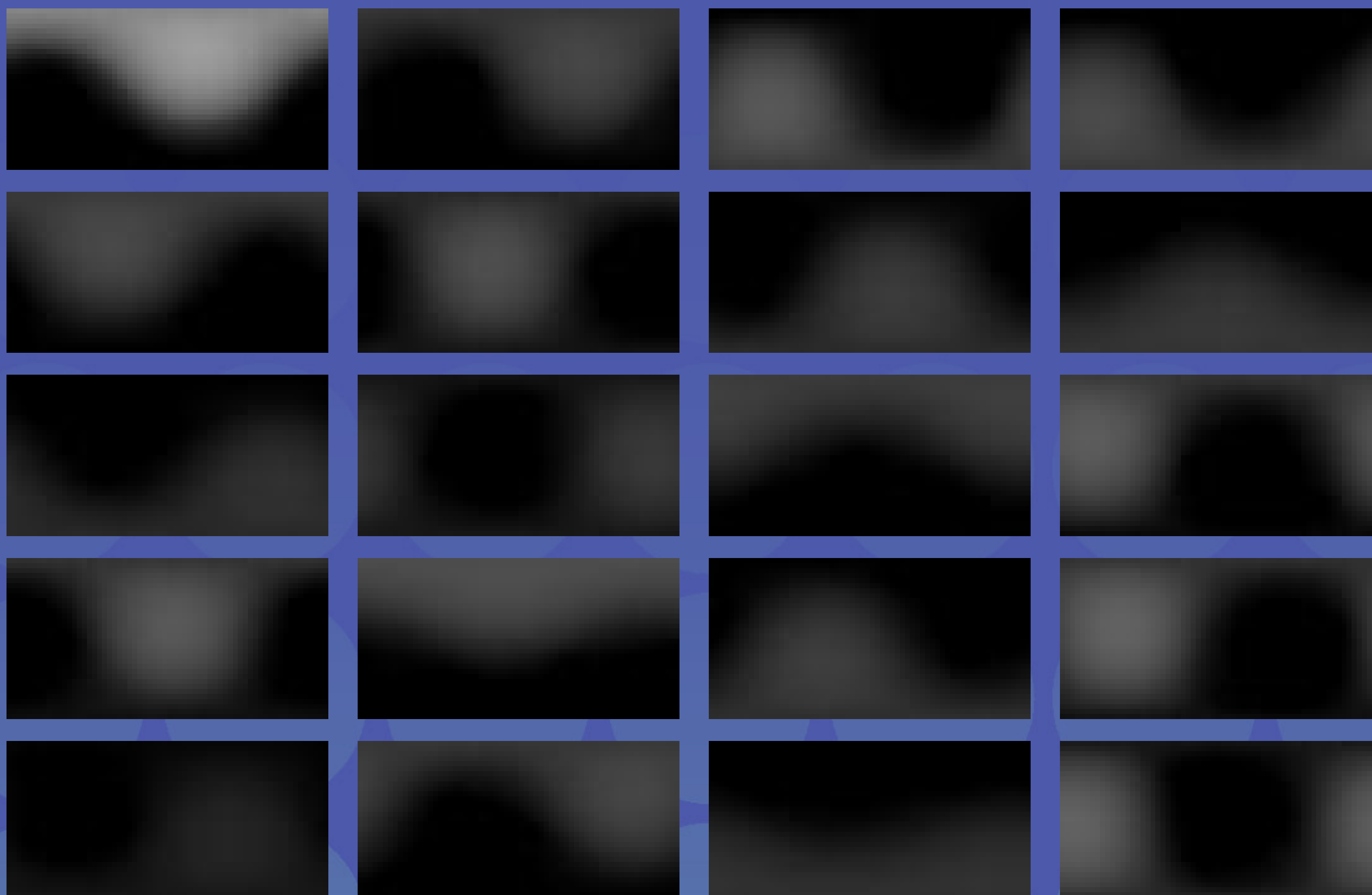


Mask to Triangle



Convolve

20 Irradiance Maps



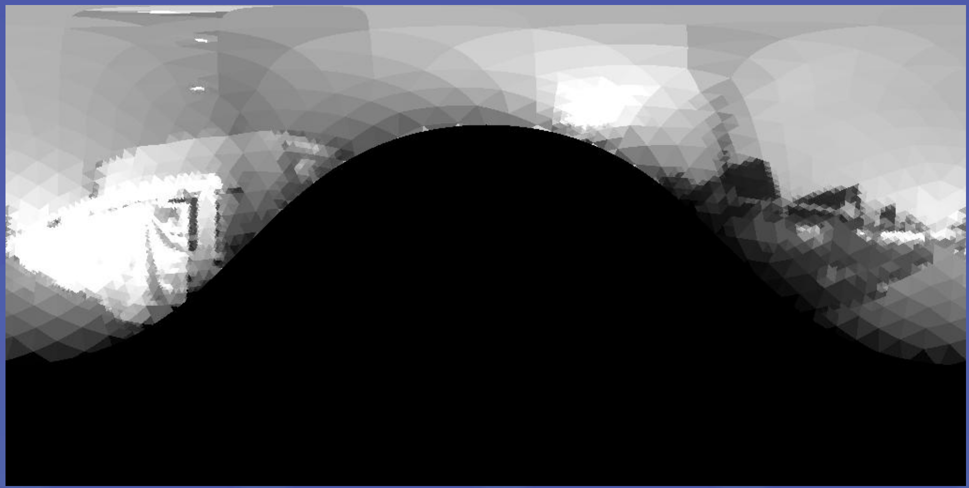
Evaluating the Integral

For surface normal n

For each triangle:

Lookup energy based on surface normal

Apply this value over triangle to get $\hat{H}$ ($\hat{H}$ is piecewise constant)



For N samples:

Select random ray direction $\omega_i \sim \hat{H}$

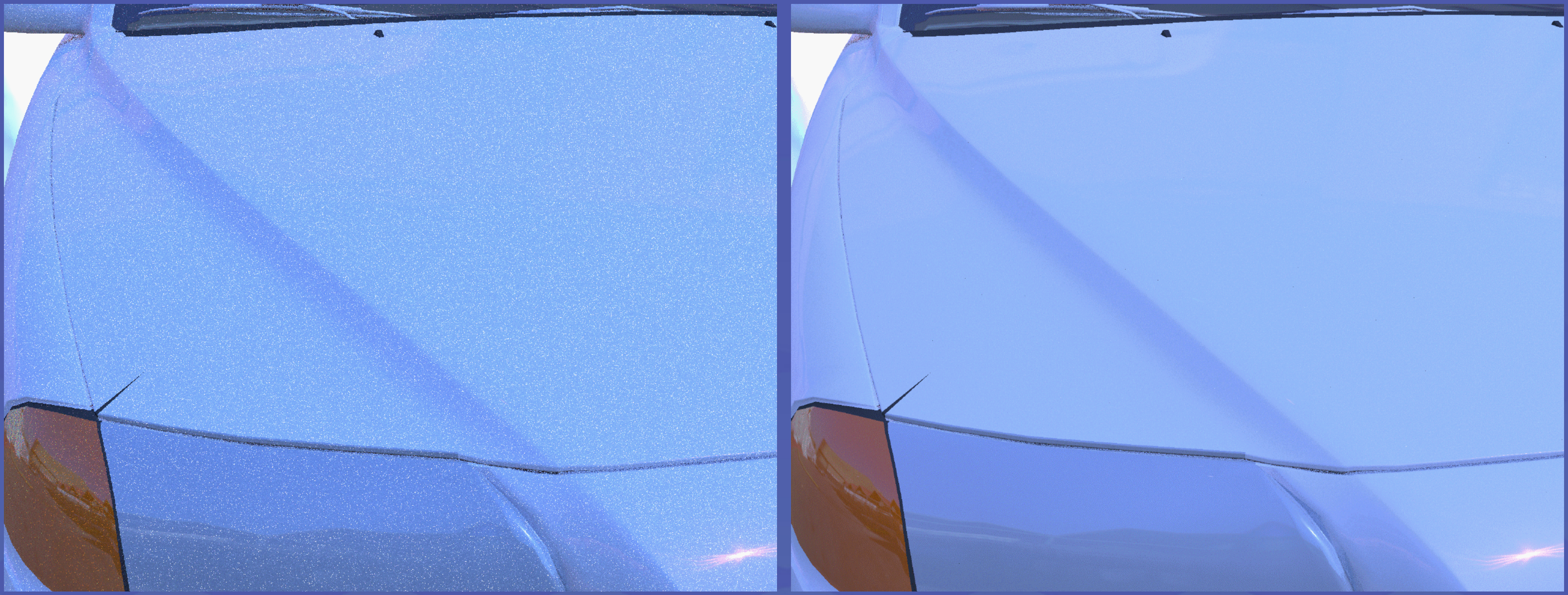
Evaluate contribution in direction ω_i (ray casting)

Result is unbiased Monte Carlo estimation.

Shadow Cache

- Kind of like irradiance cache, but stores visibility info
- Principle: Any improvements to $\hat{H}$ aid convergence
- Improves noise by 10-20 %
- See tech report for details.

Results



50 Rays per sample

Conclusion

- Converges faster - aymptotically same, constant is order of magnitude better
- Adaptive tesselation is far better than mercator projection
- Overhead per pixel is high, want to bring it down
- Unbiased scheme means only artifact is noise
- Only noise is due to visibility term

Thanks

- Doug Bloom
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- R&H modeling and lighting
- Dr. Alex Keller

For more info: http://www.rhythm.com/~jcohen